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MPM 法による地球近傍小惑星の運動学的衝突挙動の解析有効性に関する研究
A numerical investigation on effectiveness of kinetic impact deflection of near-Earth asteroids with material point method

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Near-Earth asteroids (NEAs) pose a potential threat to human life and property due to their risk of collision with Earth, making NEA defense a longstanding focus of research. Kinetic impact deflection, exemplified by NASA's DART mission, has emerged as a promising method, yet its effectiveness remains sensitive to material properties and impact parameters. This study employs the Material Point Method (MPM), an integrated Lagrangian-Eulerian computational framework capable of resolving large deformations and multi-phase flow, to numerically simulate the deflection efficiency of idealized asteroids under kinetic impacts. MPM models incorporate elastic-plastic material behavior and granular flow to capture large deformations and fragmentation processes, and characterize energy dissipation in porous and solid targets. MPM results align well with hypervelocity impact experiments for crater scaling laws, validating the approach. Parametric studies are then conducted to investigate the momentum transfer efficiency for different types of asteroids, varying a range of asteroid properties and impact conditions. The findings highlight MPM's capability in resolving complex fragmentation physics, providing actionable insights for optimizing impactor design in future planetary defense missions.

Key Words: Asteroid deflection, Hypervelocity impact, Material point method, Simdroid-MPM

1. Introduction

Near-Earth Asteroids (NEAs) pose an existential threat, evidenced by historical impacts (e.g., Chicxulub) and recent airbursts (e.g., Chelyabinsk)[1-3]. While surveys like NASA's PDCO identify hazards early, developing mitigation strategies remains critical[4]. Kinetic impact deflection (KID)—validated by NASA's DART mission[5]—is a leading solution, altering trajectories via spacecraft impacts.

KID effectiveness depends on hypervelocity collision physics: asteroid composition (monolithic to rubble piles[6,7]), impact conditions, and cratering/ejecta dynamics[8]. The momentum transfer (β factor) exceeds the impactor's momentum due to ejecta recoil[9]. Predicting β requires modelling extreme deformation and ejecta flow, challenging for heterogeneous targets[10,11].

Traditional computational methods often struggle with these challenges. Lagrangian approaches (e.g., FEM) face mesh tangling in hypervelocity impacts (>10 km/s), causing premature termination. Eulerian methods (e.g., CFD) lack precise material tracking, leading to errors like 40% underestimates of crustal ejecta[12]. Advanced techniques are needed for large deformations, multi-phase interactions, and fracture mechanics.

This study uses the material point method (MPM) for KID assessment. MPM, a hybrid Lagrangian-Eulerian framework, uses material points (tracking state variables) moving through a Eulerian background grid. It handles extreme deformations, fragmentation, and complex constitutive models—features essential for modelling the diverse potential structures of NEAs and the violent impact process.

2. Fundamental theory of the material point method

MPM merges Lagrangian particle advantages (natural history tracking) with Eulerian grid benefits (avoiding mesh entanglement). The grid enables efficient momentum equation solution in the Lagrangian phase, while grid resetting in the Eulerian phase inherently manages convection. Stress updates leverage particle-level deformation gradients, ensuring accurate modelling of large deformations and complex materials. This hybrid approach makes MPM ideal for problems involving fractures, impacts, and multiphase interactions.

MPM is formulated within the framework of continuum mechanics, the momentum balance equation in the updated Lagrangian description is stated as

$$\rho \frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \nabla \cdot \boldsymbol{\sigma} + \rho \boldsymbol{g} \tag{1}$$

where $\boldsymbol{v}$ denotes the velocity vector of a particle, $\rho(\boldsymbol{x}, t)$ is the material density, $\boldsymbol{\sigma}(\boldsymbol{x}, t)$ is the Cauchy stress tensor and $\boldsymbol{g}(\boldsymbol{x}, t)$ is the body force such as gravity. The Cartesian coordinates $\boldsymbol{x}$ is defined in the computational domain and the time t varies within the interval [0, T]. The Galerkin weak form of Eq. (1) can be stated as

$$\int_\Omega \rho \mathbf{w} \cdot \frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} \mathrm{d}V + \int_\Omega \nabla \mathbf{w} : \boldsymbol{\sigma} \mathrm{d}V - \int_\Omega \mathbf{w} \cdot \rho \mathbf{g} \mathrm{d}V - \int_{\partial\Omega} \mathbf{w} \cdot \bar{\mathbf{t}} \mathrm{d}A = 0 \tag{2}$$

Since the whole continuum body is described with n_p material points, the mass density can be written as

$$\rho\left(\boldsymbol{x}, t\right) = \sum_{p=1}^{n_p} m_p \delta\left(\boldsymbol{x} - \boldsymbol{x}_p^t\right) \tag{3}$$

where m_p is the associated mass of a material point, δ is the Dirac delta function with dimension of the inverse of volume and $\boldsymbol{x}_p^t$ is the current position of material point p at time t. Substituting Eq. (3) into Eq. (2) converts the integrals to the sums of quantities evaluated at the material points, namely

$$\sum_{p=1}^{n_p} m_p \dot{\boldsymbol{v}}_p \cdot \mathbf{w}_p + \sum_{p=1}^{n_p} \frac{m_p}{\rho_p} \boldsymbol{\sigma}_p : \nabla \mathbf{w}_p - \sum_{p=1}^{n_p} m_p \boldsymbol{g}_p \cdot \mathbf{w}_p - \sum_{p=1}^{n_p} \frac{m_p}{\rho_p h} \bar{\mathbf{t}}_p \cdot \mathbf{w}_p = 0 \tag{4}$$

Stresses of a material point is updated constitutively via

$$\boldsymbol{\sigma}_p^{n+1} = \boldsymbol{\sigma}_p^n + \boldsymbol{M}(\Delta\varepsilon_p^{n+\frac{1}{2}}, \Delta\omega_p^{n+\frac{1}{2}}) \tag{5}$$

where $\boldsymbol{M}$ is a material model (e.g., neo-Hookean, Drucker-Prager). For explicit MPM algorithm, the strain and vorticity increment of a particle can be calculated by

$$\begin{cases} \Delta\varepsilon_p^{n+\frac{1}{2}} = \dfrac{1}{2} \displaystyle\sum_{I=1}^{n_g} ((\nabla N_{Ip}^n \boldsymbol{v}_I^{n+\frac{1}{2}})^{\mathrm{T}} + \nabla N_{Ip}^n \boldsymbol{v}_I^{n+\frac{1}{2}}) \Delta t^{n+\frac{1}{2}} \\ \Delta\omega_p^{n+\frac{1}{2}} = \dfrac{1}{2} \displaystyle\sum_{I=1}^{n_g} ((\nabla N_{Ip}^n \boldsymbol{v}_I^{n+\frac{1}{2}})^{\mathrm{T}} - \nabla N_{Ip}^n \boldsymbol{v}_I^{n+\frac{1}{2}}) \Delta t^{n+\frac{1}{2}} \end{cases} \tag{6}$$

Based on the three-dimensional explicit material point method hydrocode Simdroid-MPM, a typical computational cycle alternates between these steps: grid initialization, Particle-to-Grid (P2G) mapping of mass/momentum, calculations of grid-based momentum equations, nodal velocity updates, Grid-to-Particle (G2P) transfer to particles, and particle position/stress updates. The updated particle velocity and position can be obtained by

$$\begin{cases} \boldsymbol{v}_p^{n+\frac{1}{2}} = \boldsymbol{v}_p^{n-\frac{1}{2}} + \displaystyle\sum_{I=1}^{n_I} \frac{\boldsymbol{f}_I^n}{m_I^n} N_{Ip}^n \Delta t^n \\ \boldsymbol{x}_p^{n+1} = \boldsymbol{x}_p^n + \displaystyle\sum_{I=1}^{n_I} \frac{\boldsymbol{P}_I^{n+\frac{1}{2}}}{m_I^n} N_{Ip}^n \Delta t^{n+\frac{1}{2}} \end{cases} \tag{7}$$

3. Results and discussions

3・1 Validation of MPM results

To validate the performance of the aforementioned MPM scheme in the prediction of liquid slamming, an aluminum sphere impacting onto a basalt cylinder is first simulated. In Liu's experiment[9]， as shown in Figure 1, the sphere with a diameter of 6 mm was launched at an initial impact velocity 3.9 km/s.

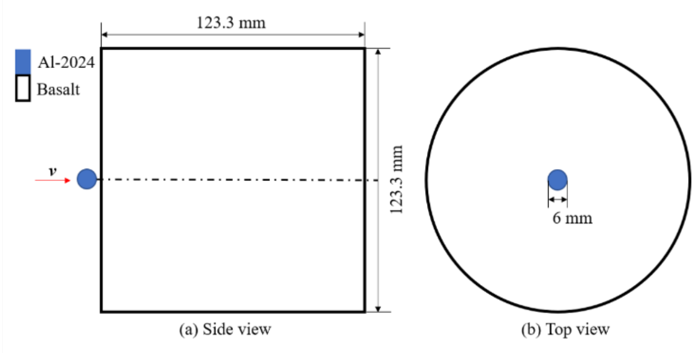

Figure 1 Sketch of the numerical model from experiment[9]

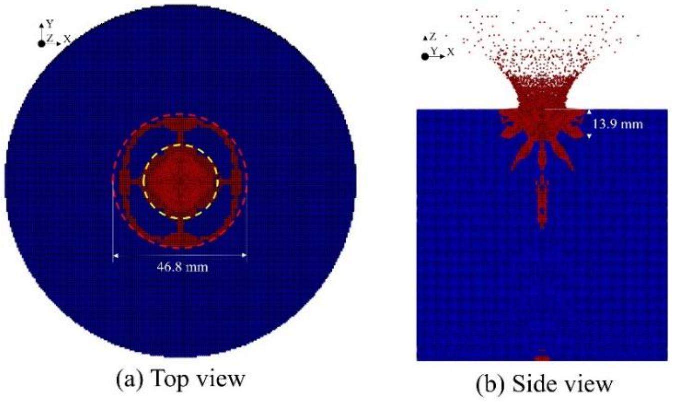

Figure 2 MPM result of craters formed in basalt, with impact velocity of 3.9 km/s.

Table 1 Comparison between numerical simulation and experimental results[9]

	Experiment	MPM Result	Error
Diameter (mm)	55	46.8	-14.91%
Depth (mm)	13.5	13.9	+ 2.96%

3・2 Kinetic impact deflection test for a small size asteroid

To further investigate the performance and feasibility of the MPM in complex fragmentation simulation, several impact experiments were conducted considering a range of impact velocity from 1.0 km/s to 5.0 km/s. The numerical setup and the initial configurations are shown in Figure 3.

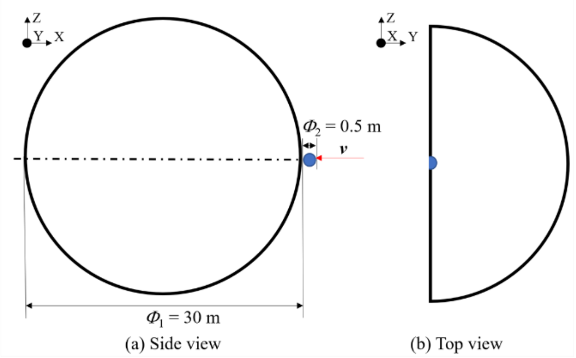

Figure 3 Variables definition for the kinetic impact deflection problem from reference[10]

It can be observed from MPM results in Figure 4 that the degree of damage and failure in the spherical rock mass significantly intensifies with increasing impact loads.

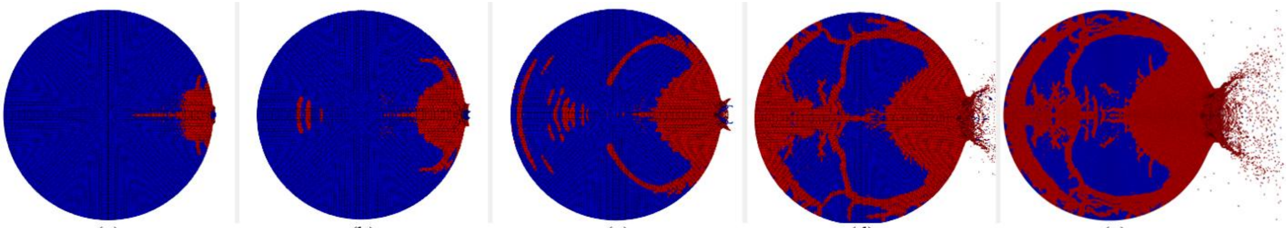

Figure 4 MPM results of impact into a spherical asteroid with different initial velocity: (a) 1.0 km/s; (b) 2.0 km/s; (c) 3.0 km/s; (d) 4.0 km/s; (e) 5.0 km/s.

4. Conclusion

This study employs Simdroid-MPM to model the effectiveness of kinetic impact deflection for NEAs. Numerical investigations demonstrate that the MPM can effectively capture large deformations and fragmentation phenomena associated with hypervelocity impact processes. The hybrid framework of MPM circumvents mesh entanglement during extreme deformations while preserving Lagrangian tracking of material history, thereby rendering it versatile for addressing complex fragmentation physics and intense multi-phase problems. The numerical results exhibit good agreement with experimental observations, verifying the feasibility and accuracy of applying MPM to evaluate and optimize impactor designs for future planetary defence missions.

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